AMC 8 / MOCK TEST 1 SOLUTIONS

1. $(B.)$ $5x + 11 = 2(x - 5) \Rightarrow 5x + 11 = 2x - 10 \Rightarrow 3x = -21 \Rightarrow \boxed{x = -7}$.

2. $(D.)$ $\dfrac{x}{y} \Rightarrow \dfrac{x-3}{y-3} = \dfrac{1}{3} \Rightarrow 3x - y = 6$ and $\dfrac{x+1}{y+1} = \dfrac{3}{5} \Rightarrow 5x - 3y = -2$. After solving those two linear equations we get $x = 5$ and $y = 9$, so the fraction is $\boxed{5/9}$.

3. $(C.)$ $50 \cdot \dfrac{20}{100} + 60 \cdot \dfrac{25}{100} = 10 + 15 = \boxed{25}$.

4. $(B.)$ *Median : List the numbers in numerical order and find the number in the middle. If there are two numbers in the middle, you average them*, so $\dfrac{130+134}{2} = \dfrac{264}{2} = \boxed{132}$.

5. $(C.)$ $\dfrac{\not 7}{6} \cdot \dfrac{\not 8}{\not 7} \cdot \dfrac{\not 9}{\not 8} \cdots \dfrac{x+1}{\not x} = 13 \Rightarrow \dfrac{x+1}{6} = 13 \Rightarrow x + 1 = 78 \Rightarrow = 77$, so $7 + 7 = \boxed{14}$

6. $(D.)$ Let's consider $\angle ABC = \alpha$, then $\angle ACB = \alpha, \angle ADC = \alpha + 25$ and $\angle AEB = 130 - \alpha$. Since $\angle BAD = 180 - \angle ABE - \angle AEB = 180 - \alpha - (130 - \alpha) = \boxed{50^o}$.

7. $(C.)$ $(x-1)^2 + 7^2 = (7-x)^2 + 5^2 \Rightarrow x^2 - 2x + 1 + 49 = 49 - 14x + x^2 + 25 \Rightarrow 12x = 24$ $\Rightarrow x = \boxed{2}$.

8. $(D.)$ Since $AB = AC = \dfrac{13}{\sqrt{2}} = \dfrac{13\sqrt{2}}{2}$, then the area of $S_1 + S_2 = 2 \cdot (\dfrac{13\sqrt{2}}{2})^2 = \boxed{169}$.

9. $(D.)$ Thickness of four papers is $2 \cdot 10^{-1} \cdot 4 = 8 \cdot 10^{-1}$, after $1st$ folding $2 \cdot 8 \cdot 10^{-1} = 16 \cdot 10^{-1}$ and after second folding $2 \cdot 16 \cdot 10^{-1} = 32 \cdot 10^{-1} = \boxed{3.2\,mm}$.

10. $(C.)$ Let's call the first two columns which have two white squares and three black squares together as "$BLOCK$". If there are 40 black squares, there are 13 $BLOCKS$ which contains 39 black squares. So we need one more half block which has two white and one black squares. Finally the number of white squares is $13 \cdot 2 + 2 = \boxed{28}$.

11. $(E.)$ For $n \in \mathbb{Z}$, $n(n+1)(n+2)(n+3) + 1 = (n^2 + 3n)(n^2 + 3n + 2) + 1 = (n^2 + 3n + 1)^2$, so if $n = 30$, we have $\sqrt{(900 + 3 \cdot 30 + 1)^2} = \boxed{991}$.

12. $(D.)$ $\sqrt{441 + 235} = \sqrt{676} = \sqrt{26^2} = \boxed{26}$.

13. $(E.)$ The area of the triangle is equal to area of the half circle with radius r. $\dfrac{6 \cdot 2r}{2} = \dfrac{\pi \cdot r^2}{2}$ $\Rightarrow \boxed{r = \dfrac{12}{\pi}}$.

14. $(D.)$ If a is between -12 and 3, then $a + 12 = 2(3 - a) \Rightarrow a = -2$. If a is greater than 3, then $a + 12 = 2(a - 3) \Rightarrow a = 18$. $18 + (-2) = \boxed{16}$.

15. $(C.)$ There are 250 three-digit numbers from 100 to 249. If we subtract the number of numbers that have all three digits repeating which are 111, 222, 333, we will find the number of numbers which are at most two digits repeating. $250 - 3 = \boxed{247}$.

16. $(B.)$ Since $DB = \sqrt{2^2+4^2} = \sqrt{20} = 2\sqrt{5}$ and $LD = 4$, then $x = \sqrt{4^2+(2\sqrt{5})^2} = \sqrt{16+20} = \sqrt{36} = \boxed{6}$.

17. $(E.)$ $a^2 - ac - ab + bc = a(a-c) - b(a-c) = (a-b)(a-c)$, on the other hand $a - b + b - c = a - c = 12$, so $(a-b)(a-c) = 8 \cdot 12 = \boxed{96}$.

18. $(D.)$ The given expression in modulo 10 will be the answer. $A = 2! + 4! + 6! \cdots + 2016! \equiv 26 \equiv 6 \pmod{10}$, because $6!, 8! \cdots 2016!$ have 2 and 5 as factors, so unit digit of those numbers are 0. $(A+1)^{A+2019} = (7)^A 7^{2019} = 7^{2!+4!+6!\cdots+2016!}(7^4)^{504}$. Since $7^4 \equiv 1 \pmod{10}$, then $7^{2!+4!+6!\cdots+2016!}(7^4)^{504} \equiv 7^2 \underbrace{7^{4!+6!+\cdots+2016!}}_{1} 1^{504} \pmod{10} \equiv 7 \cdot 1 \cdot 1 \equiv 49 \equiv \boxed{9} \pmod{10}$.

19. $(C.)$ If $\gcd(A,B) = 2^2 3^2 5$, then $n = 2, m = 2$ and $k = 1$. $m + n + k = 2 + 2 + 1 = \boxed{5}$.

20. $(C.)$ All conditions are $C(8,3)$. The number of ways we can choose the first person is $C(4,1)$. Second person can only be his or her spouse so there is only one way and the third person can be chosen in 6 ways, so the probability that two of them is married couple is $\dfrac{C(4,1) \cdot C(6,1)}{C(8,3)} = \boxed{\dfrac{3}{7}}$.

21. $(C.)$ First connect points E, D and points E, B, then connect points E, F where F is the midpoint of DB. Since $EC = EF = FD = BF$ we get $\angle EFC = \angle ECF$ and $\angle FEB = \angle FBE = \angle EAB = \boxed{25^o}$.

22. $(A.)$ $\dfrac{x+y+z-z}{z+x} + \dfrac{z+y+y-y}{z-y} = 4 \Rightarrow \dfrac{\cancel{z+x}}{\cancel{z+x}} + \dfrac{y-z}{z+x} + \dfrac{\cancel{z-y}}{\cancel{z-y}} + \dfrac{2y}{z-y} = 4 \Rightarrow 1 + \dfrac{y-z}{z+x} + 1 + \dfrac{2y}{z-y} = 4 \Rightarrow \dfrac{y-z}{z+x} + \dfrac{2y}{z-y} = \boxed{2}$.

23. $(A.)$ $b = 3^{1336} = 9^{668} > 8^{668} = 2^{2004} > 2^{2002} = a$ and $a = 2^{2002} = 128^{286} > 125^{286} = 5^{858} > 5^{857} = c$, so $\boxed{b > a > c}$.

24. $(C.)$ $\dfrac{|x|}{x} = \pm 1$, so the given sum can be $1+1+1, 1-1+1, 1-1-1, -1-1-1$ that is $3, 1, -1, 3$, $\boxed{4}$ different values.

25. $(A.)$ In one hour three pipes can fill only $\dfrac{1}{12}$ of a pool and the drain can empty only $\dfrac{1}{18}$ of the same pool. When three pipes and the drain are open, in x hours $\dfrac{1}{4}$th of the pool will be full so $\dfrac{x}{12} - \dfrac{x}{18} = \dfrac{1}{4}$ that is $x = \boxed{9}$.

AMC 8 / MOCK TEST 2 SOLUTIONS

1. $(C.)$ $5x+4x=108 \Rightarrow 9x=108 \Rightarrow x=12 \Rightarrow 2\cdot 12=\boxed{24}$.

2. $(E.)$ $\frac{2x}{5}+\frac{2}{5}=2x+\frac{1}{5} \Rightarrow \frac{2}{5}-\frac{1}{5}=2x-\frac{2x}{5} \Rightarrow \frac{1}{5}=\frac{8x}{5} \Rightarrow x=\boxed{\frac{1}{8}}$.

3. $(A.)$ $\frac{64}{4}=16$ unit squares should be shaded.10 of the unit squares are already shaded, so $16-10=\boxed{6}$.

4. $(B.)$ Let A be the sum of the ages of 5 students. If $\frac{A}{5}=8$, then $A=40$. After another student joins the group, the arithmetic mean of six students is $\frac{40+14}{6}=\frac{54}{6}=\boxed{9}$.

5. $(C.)$ $\frac{1}{\frac{1}{2}}-\frac{\frac{1}{2}}{3}=1\cdot\frac{2}{1}-\frac{1}{2}\cdot\frac{1}{3}=2-\frac{1}{6}=\boxed{\frac{11}{6}}$

6. $(D.)$ Assume $\angle ABD=\angle DBC=a$ and $\angle DCE=\angle ECB=b$, then $a+2b=90$ and $2a+2b=130$, so $a=40^o$ and $b=25^o$. $x=180-2a-b=180-80-25=\boxed{75^o}$.

7. $(E.)$ If $2x=3y$, then $x=3k$ and $y=2k$ where $k\in\mathbb{Z}$ and $x+y=3k+2k=5k$, so $x+y$ is divisible by 5. Answer is $\boxed{-10}$.

8. $(B.)$ The number of cups of orange is $\frac{750}{2}=250$. The total amount of orange juice in those cups is $1,000\cdot 0.05=50$ liters. So one cup gets $\frac{50}{250}=\boxed{0.2}$ liter orange juice.

9. $(D.)$ Because DH is median $BH=HC=HD=5$. By the Pythagorean theorem we get $AH=12$. $[ABC]=\frac{BD\cdot AC}{2}=\frac{AH\cdot BC}{2} \Rightarrow BD\cdot 13=12\cdot 10 \Rightarrow BD=\boxed{\frac{12}{13}}$.

10. $(B.)$ $1st, 2nd, 3rd, \cdots$ rows have sums $1^3, 2^3, 3^3, \cdots$ respectively. Since $343=7^3$, it is the $\boxed{7th}$ row.

11. $(E.)$ Set $2^5=x$, then $\sqrt{\frac{x^2+2x-3}{31}}=\sqrt{\frac{(x+3)(x-1)}{31}}=\sqrt{\frac{35\cdot 31}{31}}=\boxed{\sqrt{35}}$.

12. $(D.)$ If Zach can do in 36 days, then Anna can do the same job in 12 days and she can do the one-third of the same job in $\boxed{4}$ days.

13. $(E.)$ Since $|x+2|=77!$, we get two solutions from $x+2=77!$ and $x+2=-77!$, so $-77!-2+77!-2=\boxed{-4}$.

14. $(D.)$ Let E be on BD such that AE is an altitude of the side BC. Because of the $30^o-60^o-90^o$ triangle $AE=\frac{12}{2}=6$. $[ABD]=\frac{AE\cdot BD}{2}=\frac{6\cdot 9}{2}\frac{54}{2}=\boxed{27}$.

15. $(E.)$ Number of all three-digit numbers is 900. Number of three-digit numbers where no digit repeating is $9\cdot 9\cdot 8=648$. All the three-digit numbers in the form of $11x, 1x1, x11$ where $x\in\{2,3,5,7\}$ are numbers such that product of their digits is prime. So we have $4\cdot 3=12$ of these numbers. The answer is $900-648-12=\boxed{240}$.

16. $(C.)$ By similarity, $D=(3,0)$ and $A=(0,4)$ so $OD=3$, $AO=4$ and $AD=5$. Area of $ABCD$ is $5^2=\boxed{25}$.

17. (B.) The radii of the two circles are 2 and 6, the width and the height of the rectangle is 8 and 6 respectively. The area of shaded region is $[ABCD] - \frac{\pi \cdot 2^2}{4} - \frac{\pi \cdot 6^2}{4} = 6 \cdot 8 - \pi - 9\pi = \boxed{48 - 9\pi}$.

18. (E.) Since $3^4 \equiv 1 \pmod 5$, $3^{4m+10} \equiv (3^4)^m \cdot (3^4)^2 \cdot 3^2 \equiv 1^m \cdot 1^2 \cdot 9 \equiv \boxed{4} \pmod 5$.

19. (C.) If $\gcd(A, B) = 15$, then the minimum possible values of A, B are $15, 30$. $(A+B)_{min} = 15 + 30 = \boxed{45}$.

20. (D.) The equation $x + y = 12$ has $C(12, 1)$ solutions in positive integers and there is only one pair of solutions that gives $5 \cdot 7 = 35$. The probability is $\boxed{\dfrac{1}{12}}$.

21. (B.) If the hypotenuse of the $15^o, 75^o, 90^o$ triangle is 4, the altitude from the right angle is 1. $[OABC] = 2 \cdot \frac{1 \cdot 4}{2} = 4$. Since the radius of the given quarter circle is $r = OB = AC = 4$, the area of the quarter circle is $\frac{\pi \cdot 4^2}{4} = 4\pi$. The area of the shaded region is $\boxed{4\pi - 4}$.

22. (C.) Set $y = 2004$, we have $A = \frac{y}{y+1} + \frac{y}{y-1} = \frac{y(y-1)+y(y+1)}{y^2-1} = \frac{2y^2}{y^2-1}$. Since $y^2 = 2004^2$, $2y^2 = Ay^2 - A \Rightarrow y^2 = 2004^2 = \boxed{\dfrac{A}{A-2}}$.

23. (E.) If $3^{2a} = 3^{3b} = 3^{4c}$, then $2a = 3b = 4c = k$, where $k \in \mathbb{Z}$. Since a, b, c are negative real numbers then we have $a = \frac{k}{2}, b = \frac{k}{3}$ and $c = \frac{k}{4} \Rightarrow \boxed{c > b > a}$.

24. (E.) $a(x - a) + b(y - x) + c(c - y) = 0 \Rightarrow a(y - c) + b(c - a) - c(y - c) = 0 \Rightarrow (y - c)(a - c) - b(a - c) \Rightarrow (a - c)(y - c - b) = 0 \Rightarrow a = c$ or $y = c + b$. After substituting y into the given expression we get $x = a - c - c - b = \boxed{a - b}$.

25. (C.) Since $BA^2 = BE \cdot BC$, then BA is tangent to the circumscribed circle of $\triangle AEC$ at A, so $\angle BAE = \frac{\overset{\frown}{AE}}{2} = \frac{2\angle ACB}{2} = \frac{2 \cdot 20}{2} = \frac{40}{2} = \boxed{20^o}$.

AMC 8 / MOCK TEST 3 SOLUTIONS

1. (C.) Since $l = 2s - 6$ and $l - s = 4$, we have $l = s + 4$. After submitting l into the first equation, we have $s + 4 = 2s - 6 \Rightarrow s = \boxed{10}$.

2. (A.) Let's say James reads x pages on first day, then the rest of the days will be in the table below.

DAY	1	2	3	4	5	6	7
PAGES	x	x+12	x+24	x+36	x+48	x+60	x+72

The total number of pages of the book is $7x + 252$. At the end of the $4th$ day he read $\frac{3}{7}\cdot(7x+252) = 4x+72$ pages. $3x+108 = 4x+72 \Rightarrow x = 36 \Rightarrow 7x+252 = 7\cdot36+252 = \boxed{504}$.

3. (A.) If there is no questions left blank, then the student answered 20% of the 60 problems incorrectly which is $\frac{20\cdot60}{100} = \boxed{12}$.

4. (B.) $24 = \frac{8\times AB}{2} \Rightarrow AB = 6$. $(-5, 0)$ is the midpoint of AB. $x-$coordinate of B is $-5 + 3 = \boxed{-2}$.

5. (E.) Since $\angle BAC = \angle AED = 40^o$, we get $\angle BDE = \angle BED = 80^o$ and $\angle BEC = \angle ECB = 180^o - 80^o - 40^o = 60^o$, so $\triangle EBC$ is equilateral and $\alpha = \boxed{60^o}$.

6. (D.) $\frac{4}{7} + \frac{5}{11} + \frac{4}{13} = 1 - \frac{3}{7} + 1 - \frac{6}{11} + 1 - \frac{9}{13} = \boxed{3 - 3x}$.

7. (E.) Since $\triangle DHC \simeq \triangle BAC$, we get $\frac{HC}{AC} = \frac{DC}{BC}$ where $AC = 2DC \Rightarrow \frac{3}{2DC} = \frac{DC}{18} \Rightarrow DC = 3\sqrt{3}$. If $DC = 3\sqrt{3}$, then $DH = 3\sqrt{2}$ and $x = \boxed{9\sqrt{3}}$.

8. (B.) If $\frac{x-y}{y} = 5$, then $x = 6y$. Therefore $\frac{x+y}{y} = \frac{6y+y}{6y} = \boxed{\frac{7}{6}}$.

9. (A.) $\frac{1}{\sqrt[4]{17+2\sqrt{72}}} = \frac{1}{\sqrt{\sqrt{17+2\sqrt{72}}}} = \frac{1}{\sqrt{3+2\sqrt{2}}} = \frac{1}{\sqrt{2}+1} = \boxed{\sqrt{2} - 1}$.

10. (B.) The hexagon which contains 4 and ? will have $4, 1, 2, 6, 5, 3$ in clockwise direction. Therefore the answer is $\boxed{2}$.

11. (D.) The distance between two cars in 5 hours is $85 \cdot 5 - 70 \cdot 5 = 425 - 350 = \boxed{75}$ miles.

12. (D.) Total time to finish the book is $165 + 195 + 170 + 210 + 145 = 885$ minutes and the daily average $\frac{885}{5} = 177$ minutes which is $\boxed{2\ hours\ and\ 57\ minutes}$.

13. (C.) If $5^x = 1903$, then $4 < x < 5$. Thus $|x - 4| + |x - 5| = x - 4 - x + 5 = \boxed{1}$.

14. (E.) Since $BD = BC \Rightarrow 10^2 + 5^2 - 2 \cdot 10 \cdot 5 \cdot \cos\angle BAC = 10^2 + 11^2 - 2 \cdot 10 \cdot 11 \cdot \cos\angle BAC$ $\Rightarrow 120 \cos\angle BAC = 96 \Rightarrow \cos\angle BAC = \frac{4}{5}$, so the area of $\triangle ABC$ is $\frac{1}{2} \cdot 5 \cdot 10 \cdot \sin\angle BAC$ which is $\frac{1}{2} \cdot 5 \cdot 10 \cdot \frac{3}{5} = \boxed{15}$.

15. (E.) B_1, B_2, B_3 and B_4 can sit in $(4 - 1)! = 3! = 6$ ways, then one of the girls can sit in 4 ways and the second girl can sit only in one way and they can also switch their seats, so $6 \cdot 4 \cdot 2 = \boxed{48}$.

16. $(D.)$ Since $m^2 - 9n^2 = 13 \Rightarrow (m - 3n)(m + 3n) = 13 \Rightarrow m - 3n = 1$ and $m + 3n = 13 \Rightarrow m = 7, n = 2 \Rightarrow m - n = 7 - 2 = \boxed{5}$.

17. $(A.)$ Since $\widehat{BD} = 2 \cdot 38 = 76^o$ and $\widehat{CE} = 32 \cdot 2 = 64^o$, then $x = \frac{76+64}{2} = \frac{140}{2} = \boxed{70^o}$.

18. $(D.)$ $37^{14} + 56^{30} + 41^{22} \equiv 2^{14} + 0^{30} + (-1)^{22} \equiv (2^3)^4 \cdot 2^2 + 1 \equiv 1 \cdot 4 + 1 \equiv \boxed{5} \pmod 7$.

19. $(D.)$ $\frac{A}{B} = \frac{6}{10} = \frac{3}{5} = \frac{63}{105} \Rightarrow A + B = 63 + 105 = \boxed{168}$.

20. $(A.)$ $x \cdot y = 2 \cdot 111 \cdots 1 \cdot 4 \cdot 111 \cdots 1 = 8 \cdot 111 \cdots 1^2 = 8 \cdot (\frac{999 \cdots 9}{9})^2 = \boxed{\dfrac{8}{81} \cdot (10^{25} - 1)^2}$.

21. $(D.)$ Number of terms in the expansion of $(x + y + z)^n$ is $\frac{(n+1)(n+2)}{2} \Rightarrow \frac{5 \cdot 6}{2} = \boxed{15}$.

22. $(D.)$ We can consider $3 + x + y + z = 15$ and

3	x	y	z	3	x	y	z	3	x

. Therefore, we get $15 + 15 + 3 + x = 33 + x$. To have the maximum possible value, we have to choose x as 9, so $33 + 9 = \boxed{42}$.

23. $(B.)$ We are looking for $m \in \mathbb{Z}^+$ such that $285 = m \cdot k + 5$ where k is a positive integer, so $m \cdot k = 280 = 2^3 \cdot 5 \cdot 7$. The number of positive divisors of 280 is $4 \cdot 2 \cdot 2 = 16$ but $1, 2, 4, 5$ do not work. Therefore we have $16 - 4 = \boxed{12}$ positive integers.

24. $(B.)$ Since $AB = 2BC$, we get $\angle BAC = 30^o, \angle ABC = 60^o$ and $\angle EBC = 60 - 15 = 45^o$, so $\triangle BCE$ is an isosceles with $BC = CE$. If FC is connected, then $\triangle BFC$ is equilateral, $CF = CE$ and $\angle CFE = \angle CEF = 75^o$, therefore $\angle AEF + 75 = 180 \Rightarrow \angle AEF = \boxed{105^o}$.

25. $(D.)$ Let a be the amount of food for one student, then in one day 30 students can eat $30 \cdot a$ and in 25 days 30 students can eat $30 \cdot 25 \cdot a = 750a$. After x days 10 students leave that is in x days 30 students can eat $x \cdot 30a$. The remaining food amount is $750 \cdot a - 30 \cdot x \cdot a = 20 \cdot 3 \cdot a$ will be enough for three more days for 20 students. The solution is $\boxed{23}$.

AMC 8 / MOCK TEST 4 SOLUTIONS

1. (A.) The amount of handshakes equals the number of people times the number of people minus 1 divided by two, so $\frac{8\cdot 7}{2} = \frac{56}{2} = \boxed{28}$.

2. (E.) The amounts of money she spent in three days are $\$X, \$3X, \$9X$ respectively. On third day she spent $\$9X$, so the fraction is $\boxed{\dfrac{9}{13}}$.

3. (C.) Let's consider $\angle ADC = \angle DAC = a$, $\angle BEA = \angle BAE = b$, then $a + b + \alpha = 180$ by $\triangle ADE$ and $a + b - \alpha = 110$ by $\angle BAC$, so $a + b = 45^o$ and $\boxed{\alpha = 35^o}$.

4. (B.) Since $a = 4, b = 7$ and $c = 15$, $\boxed{2a + b = c}$.

5. (A.) $\dfrac{\frac{3}{2} - \frac{1}{2}}{\frac{4}{3} + \frac{8}{3}} = \dfrac{\frac{2}{2}}{\frac{12}{3}} = \boxed{\dfrac{1}{4}}$.

6. (C.) Sale price is $1200 + 1200 \cdot \frac{18}{100} = 1200 + 216 = \boxed{\$1416}$.

7. (C.) Coordinate of intersection point of given linear equations is $\left(-\frac{31}{10}, -\frac{3}{10}\right)$, it is $y = \boxed{-\dfrac{3}{10}}$.

8. (B.) If $AB^2 = BH \cdot BC \Rightarrow (3\sqrt{3})^2 = BH(BH + 6) \Rightarrow 27 = 3(3 + 6)$, so $BH = 3$. $AH^2 = BH \cdot HC = 3 \cdot 6 = 18 \Rightarrow AH = 3\sqrt{2}$. $AC^2 = AH^2 + HC^2 \Rightarrow AC^2 = (3\sqrt{2})^2 + 6^2 = 18 + 36 = 54 \Rightarrow AC = \boxed{3\sqrt{6}}$.

9. (A.) Since $87^5 \equiv -12^5, 86^2 \equiv -13^5 \cdots 45^5 \equiv -44^5 \pmod{99}$, then the sum is $\boxed{0}$.

10. (C.) $A = (3 \cdot 1) \cdot (3 \cdot 2) \cdot (3 \cdot 3) \cdot (3 \cdot 4) \cdots (3 \cdot 14) \cdot (3 \cdot 15) = \boxed{3^{15} \cdot 15!}$.

11. (C.) $\sqrt[4]{2\sqrt[3]{2\sqrt{x}}} = \sqrt[6]{2\sqrt{2}} \Rightarrow \sqrt[24]{2^{3\cdot 2} \cdot 2^2 \cdot x} = \sqrt[12]{2^2 \cdot 2} \Rightarrow \sqrt[24]{2^8 \cdot x} = \sqrt[12]{2^3} \Rightarrow \sqrt[24]{2^8 \cdot x} = \sqrt[24]{2^6} \Rightarrow 2^8 \cdot x = 2^6 \Rightarrow x = \boxed{\dfrac{1}{4}}$.

12. (D.) John's age, his dad's age and his mom's age are $x - 24, x, 2x - 48$ respectively. 5 years ago their ages were $x - 29, x - 5$ and $4x - 53$ respectively. If $x - 29 = \dfrac{x + 2x - 48}{6}$ $\Rightarrow x = 42$, so John's current age is $42 - 24 = \boxed{18}$.

13. (B.) If $x < 1$, then $x > 1$. If $1 < x < 4$, then there is no x that satisfies the given inequality. If $1 < x < 4$, then $x < 5$, so $x \in \{1, 2, 3, 4\} \Rightarrow 1 + 2 + 3 + 4 = \boxed{10}$.

14. (C.) $[AEC] = \dfrac{2 \cdot 5}{2} = 5$ and $[BEC] = \dfrac{4 \cdot 5}{2} = 10$, so the area of the shaded region is $5 + 10 = \boxed{15}$.

15. (A.) Without any restrictions 6 people can sit at a round table in $(6 - 1)! = 5!$. *Sean* and *Luke* can sit together in $2 \cdot 4!$, so $5! - 2 \cdot 4! = 4!(5 - 2) = \boxed{4! \cdot 3}$.

16. $(A.)$ $\dfrac{(72-23)(72+23)}{(34-15)(34+15)} = \dfrac{49\cdot 95}{19\cdot 49} = \boxed{5}$.

17. $(D.)$ Since $BD = FB = 6$ and $DC = CE = 7$, $AF = AE = \dfrac{36-14-12}{2} = \dfrac{10}{2} = \boxed{5}$.

18. $(E.)$ If the first day at library was $Wednesday$, the $19th$ visit will be in $18\cdot 5 = 90$ days. Since $90 \equiv 6 \pmod 7$, then the $19th$ visit will be on $\boxed{Tuesday}$.

19. $(E.)$ If $A = 30$, then $B = 60$, so $(A+B)_{max} = \boxed{90}$.

20. $(C.)$ $(2x - \dfrac{y}{2})^n = \cdots + k\cdot x^3\cdot y^4 + \cdots \Rightarrow n = 3+4$ and $k = C(7,4)\cdot 2^3\cdot \dfrac{1}{2^4} = \dfrac{35}{2} \Rightarrow$ $n+k = \dfrac{35}{2} + 7 = \boxed{\dfrac{49}{2}}$.

21. $(A.)$ One of the interior angles of a regular octagon is 135^o. If the side length of the octagon is $\sqrt{2}$, then the altitude from D to EB is 1. Using the same idea we can find $EB = 1+\sqrt{2}+1 = 2+\sqrt{2}$, therefore $S_2 = [BCSDE] = \dfrac{\sqrt{2}+2+\sqrt{2}}{2} = \dfrac{2\sqrt{2}+2}{2}$ and $S_1 = [ABE] = \dfrac{(\sqrt{2}+2)\cdot\sqrt{2}}{2} = \dfrac{2+2\sqrt{2}}{2}$, so $\dfrac{S_1}{S_2} = \boxed{1}$.

22. $(C.)$ $\boxed{74} - \boxed{63} = (2+37) - (3+7) = 39 - 10 = \boxed{29}$.

23. $(E.)$ $2^3 3^2(y-3) = (x+1)^3 \Rightarrow y-3 = 3 \Rightarrow y = 6$. $2^3 3^3 = 6^3 = (x+1)^3 \Rightarrow x+1 = 6 \Rightarrow x = 5$, so $(x+y)_{min} = 5+6 = \boxed{11}$.

24. $(E.)$ If we write the numbers in ascending order we will get the table below.

$A-B+1$	$\cdots$	$A-4$	$A-3$	$A-2$	$A-1$	A

sum of those consecutive numbers is $A\cdot B - 1 - 2 - 3 - \cdots - B + 1 = 10\cdot B \Rightarrow$ $A\cancel{B} + \dfrac{(1-B)\cdot\cancel{B}}{2} = 10\cdot\cancel{B} \Rightarrow A + \dfrac{1-B}{2} = 10 \Rightarrow 2A - B = 19 \Rightarrow 2A + 2B = 19 + 3B$ $\Rightarrow A+B = \boxed{\dfrac{19+3B}{2}}$.

25. $(A.)$ $x^2 < x \Rightarrow 0 < x < 1$. Set $x = \dfrac{1}{2}$, then $a = 4, b = \dfrac{1}{8}, c = \dfrac{1}{4}$, so $\boxed{b < c < a}$.

AMC 8 / MOCK TEST 5 SOLUTIONS

1. (E.) Luke pays \$4 for 320 oz and \$2 for the rest. If he pays 4 cents for each oz that means he paid 200 cents for $\frac{200}{4} = 50$ oz. Total weight is $320 + 50 = \boxed{370}$.

2. (D.) $X - \frac{91 \cdot X}{92} = \frac{91}{92} \Rightarrow \frac{92X - 91X}{\cancel{92}} = \frac{91}{\cancel{92}} \Rightarrow X = 91$. $91 = 7 \cdot 13$. The greatest prime factor is $\boxed{13}$.

3. (E.) Assume that $\angle DAE = \angle ADE = a$, then $\angle ACD = \angle DEC = 2a$ and $\angle ADB = \angle ABD = 3a$. Since $\angle BAC = 3a$, $\angle BAD = 2a$. Since the sum of the interior angles of $\triangle ABD$ is 180^o, $2a + 3a + 3a = 180 \Rightarrow 8a = 180 \Rightarrow 2a = \boxed{45^o} = \angle DEC$.

4. (C.) The "$mode$" is the value that occurs the most often, so $mode$ is 7. $\boxed{C}$ is not correct.

5. (A.) Since $max(a \cdot b) = 16 \cdot 16 = 256$ and $min(a \cdot b) = 32 \cdot 1 = 32$, then $256 - 31 = \boxed{225}$.

6. (C.) If $\sqrt{x\sqrt[3]{y\sqrt[4]{z}}} = \sqrt[24]{A}$, then $\sqrt[24]{x^{12}y^4z} = \sqrt[24]{A} \Rightarrow A = x^{12}y^4z$. The number of positive divisors of A is $(12+1)(4+1)(1+1) = 13 \cdot 5 \cdot 2 = \boxed{130}$.

7. (D.) Let $x = a = 1$ and $y = b = 25$, then $\sqrt{\frac{y+b}{x+a}} = \sqrt{\frac{25+25}{1+1}} = \sqrt{\frac{50}{2}} = \sqrt{25} = \boxed{5}$.

8. (C.) First draw the perpendicular segment from C to DA and let it intersect DA at E. Since $CE = 4$, by Pythagorean theorem we get $AD^2 = DC^2 - CE^2 \Rightarrow AD^2 = 80 - 16 \Rightarrow AD = 8$ and by similarity of $\triangle CDE$ and $\triangle AEC$ we get $CE^2 = DE \cdot EA \Rightarrow 4^2 = 8 \cdot EA(x)$ $\Rightarrow x = \boxed{2}$.

9. (B.) Let the number of tables for 3 people, 5 people and 7 people be x, y and z respectively. $x = \frac{12}{3} = 4$ and $y + z = 13$. We also have $5y + 7z = 79$. This gives us $z = 7$ and $y = 6$. $5 \cdot 6 = \boxed{30}$ students are seated on tables for 5 people.

10. (E.) It will be enough to find the unit digit of the sum $1! - 2! + 3! - 4!$, because rest of the sum has a unit digit zero. $1 - 2 + 6 - 24 = -19$, the unit digit is $\boxed{9}$.

11. (C.) Parallel lines have same slope so $-\frac{(a+2)}{2} = -\frac{2}{a-1} \Rightarrow a^2 + a - 6 = 0 \Rightarrow (a+3)(a-2) = 0 \Rightarrow a = \boxed{-3}$.

12. (C.) Sum of the exterior angles of any triangle is 360^o, so $3x + 5x + 7x = 360 \Rightarrow 15x = 360$ $\Rightarrow x = 24^o \Rightarrow$ the largest angle of the triangle is $180 - 3x = 180 - 3 \cdot 24 = \boxed{108^o}$.

13. (B.) If $x > 2$, then $\frac{2}{x-2} > \frac{1}{3} \Rightarrow 6 > x - 2 \Rightarrow x < 8$. If $x < 2$, then $\frac{2}{-x+2} > \frac{1}{3} \Rightarrow$ $6 > -x + 2 \Rightarrow x > -4$. $-3 + -2 + -1 + 0 + 1 + 2 + 3 + 4 + 5 + 6 + 7 = \boxed{22}$.

14. (D.) Since $\angle CAD = \angle CDA = 65^o$ and $\angle BAC = \angle ACB = 70^o$, $\angle BAD = \angle BAC - \angle CAD = 70 - 65 = \boxed{5^o}$.

15. (B.) First choose 3 players, then add Oliver and Daniel to those teams. 3 players can be chosen among 8 players in $\boxed{C(8,3)}$.

16. $(E.)$ $a^2 + \dfrac{4}{a^2} = (a - \dfrac{2}{a})^2 + 4 = 6^2 + 4 = \boxed{40}$.

17. $(B.)$ Since $OC \perp AC$ and $\angle OAC = 30^o$, then we get $(30^o - 60^o - 90^o)$ $\triangle AOC$ and $r = OC = \dfrac{2\sqrt{3}}{\sqrt{3}} = \boxed{2}$.

18. $(D.)$ $17^{17} \equiv 2^{17} \equiv (2^2)^8 \cdot 2 \equiv 2 \pmod 5$, so $3^n + 2 \equiv 4 \pmod 5 \Rightarrow 3^n \equiv 2 \pmod 5$. Since n is a 3-digit number and $3^4 \equiv 1 \pmod 5$, then for $n = \boxed{103}$, $(3^4)^{25} \cdot 3^3 \equiv 1^{25} \cdot 27 \equiv 1 \cdot 2 \equiv 2 \pmod 5$.

19. $(B.)$ If a and b be relatively prime positive integers, then $\gcd(a, b) = 1$ and lcm $(a, b) = a \cdot b$, so $1 + a \cdot b = 145 \Rightarrow a \cdot b = \boxed{144}$.

20. $(A.)$ If $(1 - 2x + x^2)^5 = (x - 1)^{10}$, then $C(10, 7) \cdot x^3 \cdot (-1)^7 = \boxed{-120}x^3$.

21. $(E.)$ $2AF = 5AE \Rightarrow AE = 2k, EF = 3k$. Connect F and G where G is on DB and $FG \parallel CD$, then $AE/AF = ED/FG = 2/5 \Rightarrow FG = FB = \boxed{10}$.

22. $(A.)$ If $xy = 10x + y = (2a - b)(x + y)$ and $yx = 10y + x = (2b - a)(x + y)$, then $11\cancel{(x + y)} = \cancel{(x + y)}(a + b) \Rightarrow a + b = 11 \Rightarrow max(a \cdot b) = 5 \cdot 6 = \boxed{30}$.

23. $(D.)$ Let's consider $|x - (3a + 4)| = D$, then $x_1 = D + 3a + 4$ and $x_2 = -D + 3a + 4$, so $x_1 + x_2 = a^2$ that is $6a + 8 = a^2$ and $a^2 - 6a - 8 = 0$. The sum of the possible values of a is $\boxed{6}$.

24. $(C.)$ $1200 = 2^4 \cdot 3 \cdot 5^2 = (2^2)^2 \cdot (5^2)^1 \cdot 3 \Rightarrow (2 + 1)(1 + 1) = 3 \cdot 2 = \boxed{6}$.

25. $(A.)$ $a = 0, b = 3$ and $c = 2$ satisfy. $a + b + c = 0 + 3 + 2 = \boxed{5}$.

AMC 8 / MOCK TEST 6 SOLUTIONS

1. $(E.)$ Consider there are m people in the group, then the total bill is \$10 $\cdot m$. If two of them have no money, then $m-2$ people will pay \$12 each. Since the total amount of bill will not change $10 \cdot m = (m-2) \cdot 12 \Rightarrow m = \boxed{12}$.

2. $(C.)$ Number 1 be the first person in the line and number 70 be the last person in the line. The 40th person from the front of line has number 40 and the 40th person from the end of the line has number 31, so there are $\boxed{8}$ people between number 31 and number 40.

3. $(B.)$ $X - \frac{40X}{100} - \frac{25X}{100} = 3.50 \Rightarrow X - \frac{65X}{100} = 3.50 \Rightarrow \frac{35X}{100} = 3.5 \Rightarrow 35X = 350$, $X = \boxed{\$10}$.

4. $(B.)$ Suppose her scores are a, b, c, then $\frac{a+b+c}{3} = 4$ and $a+b+c = 12$. Adding the fourth test score d the new arithmetic mean gives $\frac{12+d}{4} = 3.5$, so $12 + d = 14$ and $d = \boxed{2}$.

5. $(A.)$ Label the intersection point of BE and AD as F, then $\angle AFB = \angle EFD = 90 + \frac{\angle ECD}{2}$. From rectangle $EFDC$, we get $\angle ECD + 90 + \frac{\angle ECD}{2} + 110 + 100 = 360 \Rightarrow \frac{3\angle ECD}{2} = 60 \Rightarrow \angle ECD = \boxed{40^o}$.

6. $(D.)$ $\frac{57}{5} = 11 + \frac{2}{5} = 11 + \cfrac{1}{2 + \cfrac{1}{2}} \Rightarrow a = 11, b = 2, c = 2 \Rightarrow a + b + c = 11 + 2 + 2 = \boxed{15}$.

7. $(D.)$ Since $120 = 2^3 \cdot 3 \cdot 5$ and $\frac{120}{x}$ is a prime number, x can be $2^2 \cdot 15 = 60$, $2^3 \cdot 5 = 40$ and $2^3 \cdot 3 = 24$. Sum of those values is $60 + 40 + 24 = \boxed{124}$.

8. $(E.)$ Since $\frac{AE}{AB} = \frac{1}{4} \Rightarrow \frac{AE}{CD} = \frac{1}{4}$ and $\triangle AFE \approx \triangle CFD \Rightarrow \frac{AF}{CF} = \frac{FE}{FD} = \frac{AE}{CD} = \frac{1}{4}$. Since $[AEF] = 3$, $[DAF] = 4 \cdot 3 = 12$, $[DFC] = 4 \cdot 12 = 48$, $[ADC] = 12 + 48 = 60$ and $[ABCD] = 2 \cdot 60 = \boxed{120}$.

9. $(D.)$ Suppose there are y marbles inside the box initially. First Joel adds y more marbles and withdraws x marbles, now there are $2y - x$ marbles inside the box. Then he adds $2y - x$ marbles and withdraws x marbles, now there are $4y - 3x$, and then $8y - 7x$ and then $16y - 14x - x$ marbles now. Now there are no marbles inside the box, so $16y - 14x - x = 0$, $y = \boxed{\frac{15x}{16}}$.

10. $(D.)$ There are 10 2's, five 3's and two 5's. $12! = 2^{10} \cdot 3^5 \cdot 5^2 \cdot d$, and d can be the product of other prime numbers in 12!. There is one 7 and one 11, so $d = 7 \cdot 11 = \boxed{77}$.

11. $(A.)$ $\frac{-a}{b+1} \cdot \frac{-(b-5)}{a} = -1 \Rightarrow -b - 1 = b - 5 \Rightarrow 2b = 4 \Rightarrow b = \boxed{2}$.

12. $(B.)$ If $a = 10$ and $b = 1$, then $a - 1 = (2b+1)k \Rightarrow 9 = 3k \Rightarrow k = 3$. If $b = -1$, then $a - 1 = -3 \cdot 3 \Rightarrow a = \boxed{-8}$.

13. $(D.)$ If $|2x-4| = -(2x-4) \Rightarrow 2x-4 \leq 0 \Rightarrow x \leq 2$ that is $\boxed{(-\infty, 2]}$.

14. $(C.)$ The sum of the exterior angles of a rectangle is 360^o, so $a+b+(180-c)+d = 360$ and $a+b+d = 300-c \Rightarrow 300-c+180-c = 360 \Rightarrow 2c = 120 \Rightarrow c = \boxed{60^o}$.

15. $(C.)$ 5 boys line up in the front of the line in 5! ways, then girls line up in the back of the line in 4! ways, so the answer is $\boxed{5! \cdot 4!}$.

16. $(D.)$ $\dfrac{(x-2)^{\not 2}}{\cancel{(x-2)}(x+2)} \cdot \dfrac{\frac{x+2}{\cancel{x}}}{\frac{2-x}{\cancel{x}}} = \dfrac{\cancel{(x-2)}}{\cancel{(x+2)}} \cdot \dfrac{\cancel{(x+2)}}{\cancel{(2-x)}} = \boxed{-1}$.

17. $(B.)$ By drawing all diagonals, we split the regular hexagon into 6 equilateral triangles. Suppose all diagonals intersect at O and KL intersects BE and DA at M and N respectively. Since $\triangle EMK \approx EOF$, if $[EMK] = S$, then $[KMOF] = 3S$ and by similar idea we get $[EDLK] = 5S = 6 \Rightarrow S = \dfrac{6}{5}$. Since $[ABCDEF] = 24S$ we get $24 \cdot \dfrac{6}{5} = \boxed{\dfrac{144}{5}}$.

18. $(A.)$ If $1999 \equiv 4 \pmod 7 \Rightarrow 1999^{2020} \equiv 4^{2020} \pmod 7$. It is easy to see $4^3 \equiv 1 \pmod 7$, then $4^{2020} \equiv (4^3)^{673} \cdot 4^1 \equiv 1^{673} \cdot 4 \equiv \boxed{4}$.

19. $(C.)$ Suppose that number is A, then $A = 6x+2 = 8y+2 = 9z+2$ and $A-2 = 6x = 8y = 9z$. Since lcm $(6,8,9) = 72$, we get $A-2 = 72$ and $A = \boxed{74}$.

20. $(A.)$ $\dfrac{454545}{191919} - \dfrac{4545}{1919} = \dfrac{3^3 \cdot 5 \cdot 7 \cdot 13 \cdot 37}{3 \cdot 7 \cdot 13 \cdot 19 \cdot 37} - \dfrac{3^2 \cdot 5 \cdot 101}{19 \cdot 101} \Rightarrow \dfrac{3^2 \cdot 5}{19} - \dfrac{3^2 \cdot 5}{19} = \boxed{0}$.

21. $(C.)$ Suppose diagonals intersect at O then $[ADC] = \dfrac{DH \cdot AC}{2}$ and $[DOC] = \dfrac{[ADC]}{2}$

$[DOC] = [DGO] + [GOC] \Rightarrow [DOC] = \dfrac{DH \cdot AC}{4} = \dfrac{GE \cdot \frac{AC}{2}}{2} + \dfrac{GF \cdot \frac{AC}{2}}{2} \Rightarrow \dfrac{8}{4} = \dfrac{5}{4} + \dfrac{GF}{4}$
$\Rightarrow GF = \boxed{3}$.

22. $(D.)$ If $p = 2$, then $p+3 = 5$ is prime. If $p = 1$, then $p^2+p = 2$ is prime. If $p = 4$, then $p+19 = 23$ is prime. If $p+1$ is prime, then $p = 1$ or $p+1$ is an odd number. If $p+1$ is odd then $p+8$ is even, thus it is not prime. If $p = 1$, then $p+8 = 9$ is not prime. The answer is $\boxed{p+8}$

23. $(E.)$ If $n^2+29n = x^2$, then the discriminant of the quadratic equation $n^2+29n-x^2 = 0$ should be a perfect square too, so $841+4x^2 = m^2 \Rightarrow (m-2x)(m+2x) = 841 \Rightarrow m-2x = 1$ and $m+2x = 841 \Rightarrow m = 421$ and $x = 210$. If $n^2+29n = 210^2 = 44100$, then $(n+225)(n-196) = 0 \Rightarrow n = 196$, so the answer is $1+9+6 = \boxed{16}$.

24. $(C.)$ We can choose three spots among the six for even numbers in $C(6,3)$ ways and arrange them only in one way then we can arrange three odd numbers in 3! ways, so $C(6,3) \cdot 3! = \boxed{120}$.

25. $(B.)$ In $ABCDEF$, the longest diagonal is FC. One of the interior angles of a regular hexagon is 120^o. If $AB = AF$, then $\angle AFB = \angle FBA = 30^o$ and $\angle FBC = 90^o$. It is obvious that $\triangle FBC$ is $30^o - 60^o - 90^o$, finally $BC = 1 \Rightarrow FC = \boxed{2}$.

AMC 8 / MOCK TEST 7 SOLUTIONS

1. (B.) Suppose the number of correct and wrong answers are x and y respectively. If $3x - 2y = 15$ and $x + y = 30$, $3x + 3y = 90 \Rightarrow 3x + 3y - 3x + 2y = 90 - 15 \Rightarrow 5y = 75$ $\Rightarrow y = \boxed{15}$.

2. (D.) Suppose the distance between points A and B is x and M is the midpoint. If this distance is decreased by $3/10$ of AB then the new distance will be $x - \frac{3x}{10} = \frac{7x}{10}$. Suppose the new midpoint is M'. If $AM = \frac{x}{2}$ then $AM' = \frac{7x}{20}$ and $AM - AM' = \frac{x}{2} - \frac{7x}{20} = 15 \Rightarrow$ $\frac{3x}{20} = 15 \Rightarrow x = 100$. If 100 is decreased by 10%, then the new distance will be 90 and the midpoint will shift $\boxed{5\,cm}$.

3. (A.) $100 - (20 + 25 + 40) = 15$. If 15% of the paycheck is 210, then the whole thing is $\frac{210 \cdot 100}{15} = \boxed{1400}$.

4. (C.) $Mode$ is 20, $Median$ is 40 and $Arithmetic\ Mean$ is 40. Then $\boxed{2}$ of the statements are correct.

5. (E.) Since $18 = 2 \cdot 3^2 \Rightarrow 18^k = 2^k \cdot 3^{2k}$ and $30 = 2 \cdot 3 \cdot 5 \Rightarrow 2^k \cdot 3^{2k} = 2 \cdot 3 \cdot 5 \Rightarrow 2^{k-1} \cdot 3^{2k-1} = 5$ $\Rightarrow 2^{k+1} \cdot 3^{2k-1} = 4 \cdot (2^{k-1} \cdot 3^{2k-1}) = 4 \cdot 5 = \boxed{20}$.

6. (C.) Since $\triangle BAC$ is isosceles $\angle ABD = \angle BAD = 15^o$, $\angle ACD = 25$ and $\angle DAC = 55^o$, then $\angle ADC = 180 - (55 + 25) = \boxed{100^o}$.

7. (B.) If the smallest and the largest of the seven consecutive even integers are $x + 1$ and $3x - 1$ respectively, then $3x - 1 = x + 13 \Rightarrow x = \boxed{7}$.

8. (D.) Let $BC = CD = x$, then $BD^2 = 7^2 - x^2 = 11^2 - (2x)^2 \Rightarrow 3x^2 = 121 - 49 = 72 \Rightarrow$ $x = 2\sqrt{6} \Rightarrow BD = 2x = \boxed{4\sqrt{6}}$.

9. (C.) Suppose there are x candies, so each child gets $\frac{x}{n} = 2n + 2 = 2(n + 1)$, so number of candies in terms of number of children, $x = 2n(n + 1)$. If one more child joins the group, then each child gets $\frac{x}{n+1} = \frac{2n\cancel{(n+1)}}{\cancel{(n+1)}} = \boxed{2n}$.

10. (E.) 11! through 21! there are eleven 11s as a factor and there are two more 11s in 22! and 23!, so total number of elevens is $11 + 2 + 2 = \boxed{15}$.

11. (A.) $\sqrt[3]{\frac{15}{8} - \sqrt{\frac{\cancel{8}^4}{\cancel{3}} \cdot \frac{\cancel{3}}{\cancel{2}}}} = \sqrt[3]{\frac{15}{8} - \sqrt{4}} = \sqrt[3]{\frac{15}{8} - 2} = \sqrt[3]{-\frac{1}{8}} = \sqrt[3]{(-\frac{1}{2})^3} = \boxed{-\frac{1}{2}}$.

12. (B.) $\frac{2}{|-4|} = \frac{3}{|a|} \Rightarrow a = \boxed{-6}$.

13. (C.) If $|x+2| - 5 = 8$ or $|x+2| - 5 = -8$, then $|x+2| = 13$ or $|x+2| = -3$. If $|x+2| = 13$, then $x + 2 = 13$ or $x + 2 = -13$. Therefore $x = 11$ or $x = -15$. If $|x + 2| = -3$, then there is no solution. The solution set is $\boxed{\{-15, 11\}}$.

14. (B.) Since $\triangle ABC$ is acute, $x^2 < 11^2 + 7^2 \Rightarrow x^2 < 170 \Rightarrow x < 13$ and $|11 - 7| = 4 < x$, so $4 < x < 13$. We cant construct acute triangles when $x \in \{5, 6, 7, 8\}$, so x can have $\boxed{4}$ distinct values namely $\{9, 10, 11, 12\}$.

15. (C.) 5 books are chosen, 7 books aren't. Suppose N represents a book that is not chosen and $*$ represents a book that is chosen. $*N*N\cdots$ would be the way to arrange them. We need to pick 5 of the 8 $*'s$ and that is $\boxed{C(8,5) = C(8,3)}$.

16. (D.) $1071(1075 + 3) - (1071 + 2)1075 = \cancel{1071 \cdot 1075} + 3 \cdot 1071 - \cancel{1071 \cdot 1075} - 2 \cdot 1075 = 3 \cdot 1071 - 2(1071 + 4) = 3 \cdot 1071 - 2 \cdot 1071 - 8 = 1071 - 8 = \boxed{1063}$.

17. (A.) An interior angle of a regular pentagon is 108^o, so $\angle EAL = 108 - 90 = 18^o$. Since $\triangle EAL$ is isosceles, $\angle AEL = \angle ALE = \dfrac{180 - 18}{2} = \dfrac{162}{2} = 81^o$. Therefore $\angle LED = 108 - 81 = \boxed{27^o}$.

18. (D.) $5!, 6!, \cdot, 9!$ has only one zero, $10!, 11!, 12!, 13!, 14!$ has two zeros, $15!, 16!, 17!, 18!, 19!$ has 3 zeros and $20!$ has four zeros, then the total number of zeros is $5 \cdot 1 + 5 \cdot 2 + 5 \cdot 3 + 4 = \boxed{34}$.

19. (A.) If $K = 10x + 6 = 12y + 8$, then $K + 4 = 10(x + 1) = 12(y + 1)$. We need to find lcm $(10, 12)$ which is 60. Therefore $K + 4 = 60 \Rightarrow K = \boxed{56}$.

20. (A.) The number must be in one of these forms $3333, 3321, 3111, 3222, 2211$ and this can be written in $1 + C(4,2) \cdot 2 + C(4,1) + C(4+1) + C(4,2) = 1 + 12 + 4 + 4 + 6 = \boxed{27}$ ways.

21. (E.) Suppose CE and DG intersects at M where $\angle MCD = 30^o$, and $DM = 4$. Draw $AN \perp EF$, where N is on EF then $\angle BAN = 30^o$ and $AB || CD$. L is the midpoint of BC where $BC = 4$. $[LCD] = \dfrac{4 \cdot 4}{2} = \boxed{8}$.

22. (C.) $B = \dfrac{4 - x}{\cancel{4} - x - \cancel{4}} = \dfrac{4 - x}{-x} = \dfrac{x - 4}{x} = \boxed{\dfrac{1}{A}}$.

23. (C.) $a^2 - 8a + 16 + 4b^2 - 4b + 1 = 0 \Rightarrow (a - 4^2) + (2b - 1)^2 = 0 \Rightarrow a = 4$ and $b = \dfrac{1}{2}$. Thus $ab = 4 \cdot \dfrac{1}{2} = \boxed{2}$.

24. (B.) If $3x = 2x + 2a \Rightarrow x = 2a$ and $12a = a - y \Rightarrow y = -11a \Rightarrow \dfrac{y}{x} = \boxed{-\dfrac{11}{2}}$.

25. (E.) If $(\dfrac{5^2}{3^2})^{2x-4} < (\dfrac{3}{5})^{x-2} \Rightarrow (\dfrac{5}{3})^{4x-8} < (\dfrac{5}{3})^{2-x} \Rightarrow 4x - 8 < 2 - x \Rightarrow 5x < 10 \Rightarrow x < 2$. The maximum integer value of x is $\boxed{1}$.

AMC 8 / MOCK TEST 8 SOLUTIONS

1. (B.) Suppose the number of benches is x then $3x + 8 = 4(x - 3) \Rightarrow 3x + 8 = 4x - 12 \Rightarrow x = 20$ total number of students is $3 \cdot 20 + 8 = \boxed{68}$.

2. (C.) $x + 50 = m^2$ and $x + 90 = n^2$ where $m, n \in \mathbb{Z}$. $x + 91 - (x + 50) = n^2 - m^2 \Rightarrow (n - m)(n + m) = 41$. Since 41 is a prime number we get $n - m = 1$ and $n + m = 41$, Therefore $n = 21$ and $m = 20$. $x + 50 = 20^2 = 400 \Rightarrow x = 350$ or $x + 91 = 21^1 = 441 \Rightarrow x = 350$. The sum of the digits of 350 is $3 + 5 + 0 = \boxed{8}$.

3. (E.) There are 12 circles and 4 of them are not shaded $\dfrac{100 \cdot 4}{12} = \boxed{\dfrac{100}{3}}\%$.

4. (D.) Suppose the sum of the first four test scores is X and the fifth test score is a then $\dfrac{X}{4} = 60 \Rightarrow X = 240$. If the arithmetic mean of five test scores is 50, then $\dfrac{X + a}{5} = 50 \Rightarrow 240 + a = 250 \Rightarrow a = \boxed{10}$.

5. (C.) For $x = 1, y = 2$ and $z = 9$, $min(x \cdot y \cdot z) = 1 \cdot 2 \cdot 9 = \boxed{18}$.

6. (A.) Suppose $[AEF] = a$. Since $[AEB] = \dfrac{[ABCD]}{2} = [ADC]$, $a + z = a + x + y \Rightarrow \boxed{z = x + y}$.

7. (B.) $6! \cdot x = m^2$ where $m \in \mathbb{Z}^+$. If $2^4 \cdot 3^2 \cdot x = m^2$, then $min(x) = \boxed{5}$ and $m = 2^2 \cdot 3 \cdot 5$.

8. (C.) Let $BH = x$, then $DC = x + 2$. $AH^2 = AB^2 - BH^2 = AC^2 - CH^2 \Rightarrow 36 - x^2 = 64 - (x + 4)^2 \Rightarrow 36 - \cancel{x^2} = 64 - \cancel{x^2} - 8x - 16 \Rightarrow 8x = 64 - 16 - 36 \Rightarrow 8x = 12$ $x = \dfrac{12}{8} = \dfrac{3}{2}$. $BC = 2x + 4 = 2 \cdot \dfrac{3}{2} + 4 = \boxed{7}$.

9. (B.) After finishing $\dfrac{1}{a}$ of a job $1 - \dfrac{1}{a} = \dfrac{a - 1}{a}$ is left undone. $\dfrac{1}{b}$ of the remaining job is $\dfrac{1}{b} \cdot \dfrac{a - 1}{a} = \dfrac{a - 1}{b \cdot a}$ and $\dfrac{a - 1}{a} - \dfrac{a - 1}{b \cdot a} = \dfrac{a \cdot b - b - a + 1}{b \cdot a}$ is left undone which is $\dfrac{1}{a}$. $\dfrac{a \cdot b - b - a + 1}{b \cdot \cancel{a}} = \dfrac{1}{\cancel{a}} \Rightarrow b = \dfrac{a - 1}{a - 2}$. If $a = 3$, then $b = 2$. For other integer values of a we do not get an integer, so $a + b = 3 + 2 = \boxed{5}$.

10. (C.) Let $A = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6 + \cdots + 18 \cdot 21 \Rightarrow A - x = 1 \cdot (4 - 3) + 2 \cdot (5 - 4) + 3 \cdot (6 - 5) + \cdots + 18 \cdot (21 - 20) = \dfrac{18 \cdot 19}{2} = 171 \Rightarrow A = 171 + x$.

11. (D.) $x^{\frac{1}{3}} \cdot \cancel{2^{\frac{1}{6}}} = 3^{\frac{1}{2}} \cdot \cancel{2^{\frac{1}{6}}} \Rightarrow x = (3^{\frac{1}{2}})^3 \Rightarrow x = \sqrt{3^3} = \boxed{3\sqrt{3}}$.

12. (C.) Suppose the number of questions is X and John attempted $\dfrac{3x}{4}$ then the number of correct answers is $\dfrac{5}{8} \cdot \dfrac{3x}{4} = 105$ $\dfrac{15x}{32} = 105 \Rightarrow x = \boxed{224}$.

13. (D.) We need to find the minimum possible value of the denominator. For $x = \{1, -2, 3\}$ we get $\{\dfrac{40}{5}, \dfrac{40}{8}, \dfrac{40}{7}\}$ respectively so the largest value of the given expression is $\dfrac{40}{5} = \boxed{8}$.

14. $(B.)$ Let $\angle EFC = \alpha$, then $\angle FEC = 90 - \alpha$. $\angle AEB = \alpha \Rightarrow \angle BAE = 90 - \alpha$. Therefore $\triangle ABE \approx \triangle ECF \Rightarrow \dfrac{AB}{BE} = \dfrac{EC}{CF} \Rightarrow \dfrac{4}{2} = \dfrac{2}{CF} \Rightarrow CF = 1 \Rightarrow AE = \sqrt{20}, EF = \sqrt{5} \Rightarrow$ $[AEF] = \dfrac{\sqrt{20} \cdot \sqrt{5}}{2} = \boxed{5}$.

15. $(E.)$ One blue, one green and one red can be choosen in $C(3,1), C(3,1), C(2,1)$ ways respectively, so the probability is $\dfrac{C(3,1)C(3,1)C(2,1)}{C(8,3)} = \boxed{\dfrac{9}{28}}$.

16. $(C.)$ $2^{12} - 1 = (2^6 - 1)(2^6 + 1) = (2^3 - 1)(2^3 + 1)(65) = 7 \cdot 9 \cdot 65 = 7 \cdot 3^2 \cdot 5 \cdot 13$. $\boxed{33} = 3 \cdot 11$ cannot divide $2^{12} - 1$.

17. $(D.)$ $\dfrac{2004}{1 + x^{m-n}} + \dfrac{2004}{1 + x^{n-m}} = \dfrac{2004}{\dfrac{x^n + x^m}{x^n}} + \dfrac{2004}{\dfrac{x^n + x^m}{x^m}} = \dfrac{2004x^n + 2004x^m}{x^n + x^m} = \boxed{2004}$.

18. $(E.)$ $6 = 2 \cdot 3 \Rightarrow (n+1)(m+1) = 2 \cdot 3 \Rightarrow n = 1$ and $m = 2$, so all two digit numbers satisfying the given conditions are in $a^n b^m = ab^2$ form. For $b = 2$, a can be $\{2, 3, 5, 7, 11, 13, 17, 19, 23\}$; for $b = 3$, a can be $\{2, 3, 5, 7, 11\}$; for $b = 5$, a can be $\{2, 3\}$ and for $b = 7$; a can be $\{2\}$. If $6 = 6 \cdot 1 = (5+1)$ then $2^5 = 32$, and the total number is $\boxed{18}$.

19. $(E.)$ Number of $PERFECT$ subsets with 1 element is 100. Number of $PERFECT$ subsets with two or more elements can be found as follows. Choose two numbers from 100 numbers. Those numbers are the minimum and the maximum elements of the $PERFECT$ subsets and other numbers between them belong to that subset. The total number of $PERFECT$ subsets is $100 + 4950 = \boxed{5050}$.

20. $(D.)$ S, N, D, Y can be arranged in $4! = 24$ ways. One of the vowels can be arranged $(-S - N - D - Y-)$ in 5 ways and the second one can be arranged $(US - N - D - Y-)$ in 4 ways. Total number of arrangements is $4! \cdot 5 \cdot 4 = 24 \cdot 20 = \boxed{480}$.

21. $(A.)$ $\dfrac{-a}{b+1} \cdot \dfrac{-(b-5)}{a} = -1 \Rightarrow -b - 1 = b - 5 \Rightarrow 2b = 4 \Rightarrow b = \boxed{2}$.

22. $(A.)$ $\left(2002 + \dfrac{65}{11}\right) : \left(2002 + 1 + \dfrac{54}{11}\right) = \left(2002 + \dfrac{65}{11}\right) : \left(2002 + \dfrac{65}{11}\right) = \boxed{1}$.

23. $(A.)$ From the givens $a^2 = 2b$, $b^2 = 3a$ and $a \cdot b = 6$. Therefore $a^3 + b^3 = 2ba + 3ab = 5ab = 5 \cdot 6 = \boxed{30}$.

24. $(A.)$ $a^2 = 13 + \sqrt{160}, b^2 = 13 + \sqrt{120}$ and $c^2 = 13 + \sqrt{168}$. $\boxed{b < a < c}$.

25. $(B.)$ Since $x^2 = x - 1$, $(x^2)^8 + x - 1 + 3 = ((x-1)^2)^4 + x + 2 = (x^2 - 2x + 1)^4 + x + 2 = (x - 1 - 2x + 1)^4 + x + 2 = x^4 + x + 2 = (x-1)^2 + x + 2 = x^2 - 2x + 1 + x + 2 = x^2 - x + 3 = x - 1 - x + 3 = \boxed{2}$.

AMC 8 / MOCK TEST 9 SOLUTIONS

1. (B.) If $b^2 > 0$, then $b > 0$ and $-b-1 < 0$ so A lies in $4th$ quadrant. If $b > 0$ and $a+b < 0$, then $a < 0$. Finally $C = (a, b)$ lies in the $\boxed{\text{2nd quadrant}}$.

2. (C.) If the winner receives n cards from $n-1$ players, then $n(n-1) = 72 \Rightarrow n = \boxed{9}$.

3. (E.) $18^k = 2^k \cdot 3^{2k} = 30 \Rightarrow 2^{k+1} \cdot 3^{2k-1} = 2^k \cdot 2 \cdot 3^{2k} \cdot 3^{-1} = \underbrace{2^k \cdot 3^{2k}}_{30} \cdot 2 \cdot \frac{1}{3} = 30 \cdot 2 \cdot \frac{1}{3} = \boxed{20}$.

4. (B.) $\frac{2+4+6+8+10+12+14}{7} = \frac{56}{7} = \boxed{8}$.

5. (B.) $\underbrace{8 \div 2}_{4} + \underbrace{6 \cdot (-2)}_{-12} + \underbrace{10 \div 5}_{2} - 2 = 4 - 12 + \cancel{2} - \cancel{2} = \boxed{-8}$.

6. (B.) $\angle BKC = 90 + \frac{\angle BAC}{2} \Rightarrow 100 - 90 = \frac{\angle BAC}{2} \Rightarrow \angle BAC = \boxed{20^o}$.

7. (E.) $2x - 2 + x \geq 4x - 6 \Rightarrow 3x - 2 \geq 4x - 6 \Rightarrow x \leq 8$ there are $\boxed{8}$ positive integers.

8. (D.) Since $AD \perp DC$, $AB \perp BC$ and $BC = CD$, $AB = AD$, and $ED = 6$. $x^2 = 6^2 + 6^2$ $\Rightarrow x = \boxed{6\sqrt{2}}$.

9. (A.) He slept $x + \frac{x}{2} = \frac{3x}{2}$ hours on Sunday and he was awake for $(24-x) \cdot \frac{90}{100}$ hours on Sunday, so $\frac{3x}{2} + (24-x) \cdot \frac{90}{100} = 24 \Rightarrow \frac{3x}{2} + \frac{216-9x}{10} = 24 \Rightarrow 15x + 216 - 9x = 240 \Rightarrow 6x = 24$ $\Rightarrow x = \boxed{4}$.

10. (B.) $14! + 13! + 12! = 12!(14 \cdot 13 + 13 + 1) = 12! \cdot 196 = 12! \cdot 2^2 \cdot 7^2$. Since $39 = 3 \cdot 13$, the given expression cannot be divisible by $\boxed{39}$.

11. (C.) $\sqrt[4]{2\sqrt[3]{2\sqrt{x}}} = \sqrt[6]{2\sqrt{2}} = \sqrt[24]{2^8 \cdot x} = \sqrt[12]{2^3} \Rightarrow 2^{\frac{8}{24}} \cdot x^{\frac{1}{24}} = 2^{\frac{3}{12}} \Rightarrow x^{\frac{1}{24}} = 2^{\frac{3}{12}-\frac{8}{24}} \Rightarrow$ $x^{\frac{1}{24}} = 2^{-\frac{2}{24}} \Rightarrow \left(x^{\frac{1}{24}}\right)^{24} = \left(2^{-\frac{2}{24}}\right)^{24} \Rightarrow x = 2^{-2} \Rightarrow x = \boxed{\frac{1}{4}}$.

12. (C.) If any three of the eight points are not collinear then we could construct $C(8,3)$ triangles. If four of the points are collinear then we cannot construct $C(4,3)$ triangles with those four points so the total number of triangles is $C(8,3) - C(4,3) = \frac{8!}{5! \cdot 3!} - 4 = \boxed{52}$.

13. (B.) $|2x-1| + |3x-4|$ has a minimum value when $x = \frac{1}{2}$ or $x = \frac{4}{3}$. If $x = \frac{1}{2} \Rightarrow$ $\left|3 \cdot \frac{1}{2} - 4\right| = \left|-\frac{5}{2}\right| = \frac{5}{2}$. If $x = \frac{4}{3} \Rightarrow \left|2 \cdot \frac{4}{3} - 1\right| = \frac{5}{3}$. The minimum possible value is $\boxed{\frac{5}{3}}$.

14. (C.) Let E be the midpoint of BC then $BE = EC$ and $AD = AE = BE = EC$. Let $\angle BAE = \angle EBA = a$, then $\angle AEB = \angle EDA = 180 - 2a \Rightarrow 180 - 2a + 180 - 2a + 15 + a = 180 \Rightarrow a = 65^o$ and $\angle DCA = 90 - 65 = \boxed{25^o}$.

15. (C.) Imagine you roll one after the other. You consider a roll a success if the number that comes up is different from all the previous numbers. You start with one. This is always a success so $P(first) = 1 = 6/6$. Your second roll is a success if one of the remaining 5 numbers shows, so $P(second) = 5/6$ and so on. Since all the rolls are independent, the total probability is the product of $\frac{6}{6} \cdot \frac{5}{6} \cdot \frac{4}{6} \cdot \frac{3}{6} \cdot \frac{2}{6} \cdot \frac{2}{6} = \boxed{\frac{6!}{6^6}}$.

16. $(D.)$ Suppose the length of the stick is x. $\frac{\frac{x}{2}}{9} - \frac{\frac{x}{2}}{14} = 10 \Rightarrow \frac{x}{18} - \frac{x}{28} = 10 \Rightarrow \frac{1}{2} \cdot \left(\frac{14x-9x}{14\cdot 9}\right) = 10$ $\Rightarrow \frac{5x}{28\cdot 9} = 10 \Rightarrow \boxed{x = 504}$.

17. $(B.)$ Since $DE = BE$, $BD = BE = ED$. That means $\triangle DEB$ is equilateral. Therefore $\angle AEB = \boxed{30^o}$.

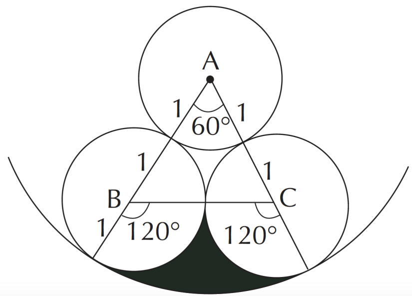
18. $(B.)$ If $23a$ is divisible by 8 then $9423a$ is divisible by 8. 232 is divisible by 8 and $a = \boxed{2}$.

19. $(B.)$ a, b can be one of $\{2, 3, 4, 5\}$, We can construct two digit numbers with distinct digits in $C(4, 2) = \boxed{6}$ ways. Suppose we choose 2 and 3. There is only one way we can construct ab and that is 23.

20. $(E.)$ Since $DK = AM = 3$, $MB = KC$ and $\triangle LKC \approx \triangle LMA$, $\frac{KL}{LM} = \frac{KC}{AM} \Rightarrow \frac{4}{2} = \frac{KC}{3} \Rightarrow$ $KC = MB = \boxed{6}$.

21. $(C.)$ Suppose A is a perfect cube, then $A = p_1^{3a_1} \cdot p_2^{3a_2} \cdots p_k^{3a_k} \Rightarrow$ the number of integer divisors of A is $2(3a_1 + 1)(3a_2 + 1) \cdots (3a_k + 1) \equiv 2 \pmod 3$. $\boxed{238} \equiv 1 \pmod 3$.

22. $(D.)$ Area of the sector of a circle with radius 3 and central angle 60^o is $\frac{60}{360} \cdot \pi \cdot 3^2 = \frac{3}{2} \cdot \pi$. Area of the sector of a circle with radius 1 and central angle 120^o is $2 \cdot \frac{120}{360} \cdot \pi \cdot 1^2 = \frac{2}{3} \cdot \pi$. We have two of these sectors. $[ABC] = \frac{2^2 \cdot \sqrt{3}}{4} = \sqrt{3}$. Finally, the area of the shaded region is $\frac{3}{2}\pi - \left(\frac{2}{3}\pi + \sqrt{3}\right) = \boxed{\frac{5}{6}\pi - \sqrt{3}}$.



23. $(D.)$ Suppose the numbers are a, b, c, d and $a \le b \le c \le d$, We need to find $c + d$. If $a + b = 1$ and $a + c = 5$, then there are two cases which are $a + d = 8$ and $b + c = 9$ or $a + d = 9$ and $b + c = 8$. For each case $a + b + c + d = 17$. Since $a + b = 1$ we get $c + d = \boxed{16}$.

24. $(E.)$ In the first two rows there are $2 \cdot n$ numbers and including the $3rd$ row we have $3 \cdot n$ numbers. $2n < 20 \le 3n \Rightarrow 7 \le n \le 9$. By using the same idea we have $4n < 41 \le 5n \Rightarrow$ $n \ge 9 \Rightarrow n = 9$ and $(m-1)n < 103 \le mn \Rightarrow 9(m-1) < 103 \le 9m \Rightarrow m = 12$. Finally, $m + n = 12 + 9 = \boxed{21}$.

25. $(B.)$ Since $(n+1)(n^2-n+1) = n^3-2n-1$, then $n^3+3 = (n+1)(n^2-n+1)+(2n+4)$, Thus $(n^2-n-1)|(2n+4)$. For $n = -2$ we have one solution, if $n \ne -2 \Rightarrow |2n+4| \ge |n^2-n-1|$, so $n \in \{-1, 0, 1, 2, 3\}$. n has $\boxed{6}$ values.

AMC 8 / MOCK TEST 10 SOLUTIONS

1. $(E.)$ $60 = 3 \cdot 20$; we got 20 sodas back. $20 = 3 \cdot 6 + 2$, we got 6 sodas back and 2 sodas not returned yet. $6 + 2 = 8 = 3 \cdot 2 + 2$, we have 2 sodas back and 2 not returned yet. $2+2 = 4 = 3 \cdot 1+1$, we have 1 soda back. Total number of sodas is $60+20+6+2+1 = \boxed{89}$.

2. $(C.)$ $n - 2 + 1 + 2n - 4 = 52 \Rightarrow 3n - 5 = 52 \Rightarrow 3n = 57 \Rightarrow n = \boxed{19}$.

3. $(C.)$ $36 - 36 \cdot \dfrac{25}{100} = 36 - 9 = \boxed{\$27}$.

4. $(E.)$ $\dfrac{X}{10} = 18 \Rightarrow X = 180 \Rightarrow \dfrac{180 + 23 + 37}{12} = \dfrac{240}{12} = \boxed{20}$. ($X$: sum of ten numbers)

5. $(A.)$ $\dfrac{a^2 - 3a - 4}{a - 4} = \dfrac{\cancel{(a-4)}(a + 1)}{\cancel{(a-4)}} = a + 1 = 111 + 1 = \boxed{112}$.

6. $(A.)$ $\angle DBC = \dfrac{\angle DBK}{2} = \dfrac{180 - 60}{2} = \dfrac{120}{2} = 60^o$. $\angle BDA = \angle DBC + \angle BCD = 60 + 25 = 85^o$. $\angle BAD = 180 - \angle BDA - \angle ABE = 180 - 85 - 60 = \boxed{35^o}$.

7. $(A.)$ Since $11! + n = n\Big(\dfrac{11!}{4} + 1\Big)$, where $n \leq 11$, none of those numbers is prime. The answer is $\boxed{0}$.

8. $(B.)$ Let $CE = x$ then $BC = 24 - x$. By Pythagorean theorem $a^2 + x^2 = 100$ and $(24 - x)^2 + 4a^2 = 400$, that is $(24 - x)^2 + 4a^2 = 4(a^2 + x^2) \Rightarrow x^2 + 16x - 8 \cdot 24 = 0$ $(x + 24)(x - 8) = 0 \Rightarrow x = 8 \Rightarrow DC = 6 \Rightarrow AC = \boxed{12}$.

9. $(B.)$ If $M \leq 2K, L - 4 \geq M$ and $K + L \leq 40$, then from $\dfrac{M}{2} \leq K$ and $M \leq L - 4$ we have $\dfrac{3M}{2} \leq K + L - 4$. On the other hand from $K + L \leq 40$ and $\dfrac{3M}{2} \leq K + L - 4$ we have $M \leq 24$, the maximum possible value of M is $\boxed{24}$.

10. $(C.)$ Since $3^8 - 1 = (3 - 1)(3 + 1)(3^2 + 1)(3^4 + 1) = 2 \cdot 4 \cdot 10 \cdot 82$, it is easy to see that 10, 16, 20 and 41 divide $3^8 - 1$, but $\boxed{18}$ cannot divide.

11. $(B.)$ Equation of the line passes through B is $y = -\dfrac{3}{5}(x - \dfrac{5}{2})$ and the coordinates of B is (a, a) where $OA = CO = a$, so $a = -\dfrac{3}{5}(a - \dfrac{5}{2}) \Rightarrow a = OC = \boxed{\dfrac{15}{24}}$

12. $(A.)$ Suppose the number of tests John has failed is x then $7x \cdot 25 - 50 \cdot x = 375 \Rightarrow 175x - 50x = 375 \Rightarrow 125x = 375 \Rightarrow x = \boxed{3}$.

13. $(C.)$ It is easy to see that $x = 2$ and $3 \cdot 2 - y = 0 \Rightarrow y = 6$. If $y - z - 4 = 0 \Rightarrow 6 - z - 4 = 0$ $\Rightarrow z = 2 \Rightarrow x + y + z = 2 + 6 + 2 = \boxed{10}$.

14. $(A.)$ Since $\angle ABC = 60^o$ and $BD = BC$, $\triangle DBC$ is equilateral so $\angle DCE = \angle CED = 40^o$, $\angle CDE = 100^o$ and $\angle ADE = \boxed{20^o}$.

15. $(D.)$ $\dfrac{10x + y + 0.1z - 10z - y - 0.1x}{x - z} = \dfrac{9.9x - 9.9z}{x - z} = \dfrac{9.9(x - z)}{x - z} = \boxed{9.9}$.

16. $(D.)$ $C(8-1, 3-1) = C(7, 2) = \boxed{21}$.

17. $(B.)$ $(5^x + 5^{-x})^3 = 6^3 \Rightarrow 125^x + 3 \cdot 5^x + 3 \cdot 5^{-x} + 125^{-x} = 196 \Rightarrow 125^x + 125^{-x} = 196 - 3 \cdot 6 = 196 - 18 = \boxed{178}$.

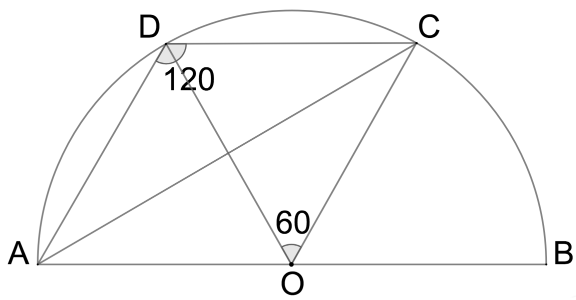
18. $(B.)$ Draw the perpendicular line segment from E to DC and AB, where H and J are the intersection points respectively. If $\dfrac{EF}{EA} = \dfrac{2}{3} \Rightarrow \dfrac{[EFH]}{[EAJ]} = \dfrac{[EHG]}{[EJB]} = \dfrac{4s}{9s}$, then $[FHJA] = [HGBJ] = 5s$ and $\dfrac{[EFH]}{[FDA]} = \dfrac{[EHG]}{[GBC]} = \dfrac{4s}{s}$ then $[FDA] = [GBC] = s$. Finally, $\dfrac{[EFG]}{[ABCD]} = \dfrac{8s}{12s} = \boxed{\dfrac{2}{3}}$.

19. $(B.)$ Since $110 = 2 \cdot 5 \cdot 11$ and 222222222222 is $0, 2, 0$ in (modula 2, 5, 11) respectively, $\boxed{22}$ satisfy those conditions.

20. $(A.)$ $p(m+n) - 2q(m+n) = (m+n)(p-2q) = 4 \cdot (-5) = \boxed{-20}$.

21. $(A.)$ Since $\dfrac{23! + 24!}{5^x} = \dfrac{25 \cdot 23!}{5^x}$ and $23 \div 5 = 4 \cdot 5 + 3$, there are four 5's in 23! and two 5's in 25. The total number of fives is $\boxed{6}$.

22. $(C.)$ Since $AO = OB = OC = DC = AD = 6$ and $\angle ADC = 120^o$, $\angle DOC = 60^o$, $[ACD] = \dfrac{1}{2} 6 \cdot 6 \cdot \sin 120 = 18 \cdot \dfrac{\sqrt{3}}{2} = 9\sqrt{3}$ and $[DOC] = \dfrac{1}{2} 6 \cdot 6 \cdot \sin 60 = 18 \cdot \dfrac{\sqrt{3}}{2} = 9\sqrt{3}$. The area of the sector with a central angle of 60^o is $\dfrac{\pi \cdot 6^2 \cdot 60}{360} = 6\pi$. Finally the area of the shaded region is $6\pi - 9\sqrt{3} + 9\sqrt{3} = \boxed{6\pi}$.



23. $(E.)$ If $x^5 = x^2 x^2 x$, then $(x+1)(x+1)x = (x^2 + 2x + 1)x = (x + 1 + 2x + 1)x = (3x + 2)x = 3x^2 + 2x = 3(x+1) + 2x = \boxed{5x + 3}$.

24. $(B.)$ $79^{9n+5} \equiv 2^{9n+5} \equiv (2^3)^{3n} \cdot 2^3 \equiv 1^{3n} \cdot 1 \equiv \boxed{1} \pmod 7$.

25. $(B.)$ $C(4, 2) \cdot C(7, 2) = 6 \cdot 21 = \boxed{126}$.