cyber math

Open Online Tournament

Sample Problems

Saturday, March 26, 2022

These are some sample problems that are meant to represent the style and general difficulty level of the problems featured in the upcoming Open Online Tournament!

Middle School Division

1. How many permutations of the word BOSTON have all letters other than the two Os in alphabetical order from left to right?

2. Kenneth's favorite number is a two-digit prime number, both of whose digits are prime, that is not a multiple of 5, and has the property that, when its digits are reversed, it is smaller, and still odd, but also not a multiple of 5. What is Kenneth's favorite number?

3. Point O inside equilateral triangle ABC is at a distance of 2 from $\overline{AB}$, $\overline{BC}$, and $\overline{CA}$. Compute the square of the area of $\triangle ABC$.

4. A smartphone lasts twice as long web browsing than it does on heavy gaming. Suppose that, beginning from a fully charged state, the phone is used for 2 hours of browsing and 1 hour of gaming, and drops to 88 percent charge after that hour. Assuming that the charge drops at a constant rate for each task, the number of hours it would last web browsing is $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

5. Compute the sum of all integers between 0 and 15, inclusive, that cannot be the remainder when a sum of two positive integer perfect squares is divided by 16.

6. Define the *oddity* of a positive integer to be the square of the number of odd digits it has. Compute the sum of the oddities of the positive integers from 1 to 100, inclusive.

7. How many triangles with integer side lengths, and positive area, have all side lengths less than or equal to 6?

8. What is the value of $\sqrt{17 \cdot 18 \cdot 19 \cdot 20 + 1}$?

9. For how many positive integers $k \leq 96$ can the fraction $\frac{k}{k+96}$ be simplified?

10. Compute the sum of the distinct prime factors of $22^4 - 20^4$.

High School Division

1. What is the remainder when $1^{2020} + 2^{2020} + 3^{2020} + \cdots + 2020^{2020}$ is divided by 2021?
2. Suppose that
$$\sum_{k=1}^{\infty}(2k+1)r^k = 3r + 5r^2 + 7r^3 + 9r^4 + \cdots = \frac{27}{25}$$
for some real number $-1 < r < 1$. The value of r can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m+n$.
3. Suppose that x and y are randomly chosen integers from 1 to 5, inclusive, with $x < y$. The expected value of $x^2 + 2y^2$ can be written in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. Compute $p+q$.
4. A positive integer is called *timid* if any two consecutive digits in the integer are either equal or only 1 apart. For example, neither 469 nor 2020 is timid, but 2123 and 4456 are. Compute the number of timid four-digit integers.
5. What is the sum of all prime numbers p such that $p+2$ is prime that satisfy the congruence $p^p + p \equiv 1 \pmod{p+2}$?
6. What is the total volume of the figure in the 3-dimensional coordinate plane containing all points that are within 1 unit of some point lying on a line segment with length 1?
7. What is the remainder when 5^{97} is divided by 23?
8. Among all permutations of the word TOURNAMENT, compute the number of times in which the substring TN appears in total.
9. What is the remainder when $123456\cdots202020212022$ is divided by 18?
10. Let r and s be the roots of $x^2 + 7x + 11$. Compute $\sqrt{r} + \sqrt{s}$.