cyber math

Open Tournament

Online

Contest File (Asia/Australia)

Middle School Division

Saturday, March 26, 2022

1. How many positive integer factors of 24 are even?
   (a) 4
   (b) 6
   (c) 7
   (d) 8
   (e) none of the above

2. At a stationery store, 3 pencils cost 70 cents, but 25 pencils cost only 470 cents. Compute the amount, in cents, one would save by buying 75 pencils in 3 packs of 25 instead of 25 packs of 3.

3. Eight fair coins are flipped at the same time. The most likely outcome has a probability $\frac{p}{q}$ of occuring, where p and q are relatively prime positive integers. Compute $p+q$.

4. Suppose that $x = -\frac{9}{2}$ is the unique real solution to the equation $x^2 + ax + b = 0$. Compute $a + b$.

5. Milvia receives the following question on her math test: "The area of a 30° sector of a circle with radius r is equal to $\frac{p}{q}\pi$ for relatively prime positive integers p and q, where q may be 1. Compute $p + q$." Given that she correctly answers 28, compute the sum of all possible positive integer values of r.
   (a) 25
   (b) 28
   (c) 30
   (d) 37
   (e) none of the above

6. How many ordered tuples (a, b, c) of positive integers satisfy $a + b + c < 10$?
   (a) 45
   (b) 56
   (c) 70
   (d) 84
   (e) none of the above

7. Triangle ABC has $AB = 13$, $BC = 14$, and $CA = 15$. Point D lies on $\overline{AB}$ with $AD = 5$, and point E lies on $\overline{CA}$ with $CE = 10$. The area of $\triangle ADE$ can be written in the form $\frac{m}{n}$, where m and n are relatively prime positive integers. Compute $m + n$.

8. For each positive integer n, let $s(n)$ be the sum of the digits of n. Compute the sum of all positive integers $n \leq 100$ such that $s(n^2) = s(n)^2$.

9. Compute the number of denominators of all fractions between $\frac{92}{99}$ and $\frac{93}{100}$ that have denominators less than or equal to 2022.

10. A mixture is 19 parts Substance A to 11 parts Substance B. If another mixture which consists of 62 percent Substance A to 38 percent Substance B is mixed well with the first mixture, in order for the new mixture to be at least $\frac{5}{8}$ Substance A by volume, the second mixture must have had total volume at most t times that of the first mixture. Compute t. Express your answer as a common fraction.

11. A rectangle has area $\frac{144}{25}$ and integer perimeter. Compute the sum of all possible values for its longer side length.

12. Lexine wants to buy exactly 20 donuts at MoreDonuts. The available flavors are vanilla, strawberry, chocolate cream, and jelly. Among the pairs $\{\text{vanilla}, \text{strawberry}\}$ and $\{\text{chocolate cream}, \text{jelly}\}$ of different donut types, Lexine wants to buy exactly 10 donuts of either of those types (so the numbers of vanilla and strawberry donuts she buys must sum up to 10, and likewise for the other pairs). In how many ways can she buy donuts? (Two ways are considered different if the number of donuts of any given type is different.)

13. Palmer starts at one of the squares of a square grid with 2 rows and 3 columns. He travels to an adjacent square that he has not already visited, and wants to eventually reach every square in the grid. In how many ways can he do this?

14. Suppose that, for a prime number p, $20p$ has n positive integer divisors. Compute the sum of the possible values of n.

15. If the real roots of $x^3 - 30x^2 - kx + 20k$ are in geometric progression for some real number k, compute k. Express your answer in simplest radical form.

16. Beginning with a 3×3 grid of squares, Rebekah selects one of the squares in each column, each with equal probability. She then crosses it, and all squares below it in its column, out with an X. She then repeats this process with any remaining squares in each column, until all squares in the grid are crossed out. Compute the expected number of squares she selects. Express your answer as a common fraction.

17. How many positive integers have base-4 representations with at most 5 digits that contain at least three of the same digit in a row?

18. The side lengths of a triangle with area 1 are 2, s, and t for some real numbers s and t with $s + t = 3$. Compute st. Express your answer as a common fraction.

19. Triangle ABC has $AB = 13$, $BC = 14$, and $CA = 15$. Let O be the circumcenter of $\triangle ABC$ and let $P \in \overline{BC}$ be the foot of the A-altitude of $\triangle ABC$. Line $\overline{OP}$ intersects $\overline{AC}$ at point D. Compute $\frac{AD}{DC}$.

20. Adele draws a rhombus in the coordinate plane with vertices at the points $(0, \pm 12)$ and $(\pm 5, 0)$. She then draws a circle centered at the origin with radius 5. The area of the quadrilateral whose vertices are the points of intersection of the rhombus and the circle can be written in the form $\frac{p}{q}$, where p and q are relatively prime positive integers. Find the sum of all prime numbers that divide either p or q.